7 Gravitational Field

Force between point masses

Newton's Law of Gravitation states that two point masses experience an attractive force which is proportional to the product of their masses and inversely proportional to the square of the distance between their centres of mass.

Gravitational force $\boldsymbol{F_G} = \frac{GMm}{r^2}$ where $G = 6.67\times10^{-11}$ N m^2 kg^{-2} (given) is the universal gravitational constant.

Note: Spherical bodies can be treated like point masses at sufficiently large distances.

Gravitational Field

A **gravitational field** is a region of space in which a mass experiences a force due to the presence of another mass.

The **gravitational field strength *g*** at a point is defined as the gravitational force per unit mass acting on a mass placed at that point.

$$g = \frac{F_G}{m} = \frac{GM}{r^2} \text{ (N kg}^{-1}\text{ or m s}^{-2}\text{)}$$

Field near surface of the Earth

g is approximately constant and is equal to **free fall acceleration**.

Gravitational Potential

The **gravitational potential energy *U*** of an object at a point in a gravitational field is defined as the work done by an external agent in bringing the object from infinity to that point without a change in KE.

Since F_G is not constant, by integration, U between 2 point masses M and m separated by a distance r is $U = -\frac{GMm}{r}$ (unit: J).

Gravitational Potential

The **gravitational potential ϕ** at a point is defined as the work done per unit mass by an external agent in bringing the mass from infinity to that point, i.e.

$\phi = \frac{U}{m} = -\frac{GM}{r}$ (unit: J kg^{-1}) (given)

The relationship between g and ϕ is $g = -\frac{d\phi}{dr}$,

Likewise, $F_G = -\frac{dU}{dr}$.

Satellite Motion

Assuming a uniform circular orbit, we can deduce that the gravitational force provides the centripetal force of a satellite, i.e. $F_G = F_C \rightarrow \frac{GMm}{r^2} = \frac{mv^2}{r} = mr\omega^2$

Linear speed $v = \sqrt{\frac{GM}{r}}$ and period $T = \sqrt{\frac{4\pi^2 r^3}{GM}}$.

Thus, Kepler's Third Law: $T^2 \propto r^3$ is useful to remember.

Geostationary Orbit

A **geostationary orbit** refers to a circular orbit around the Earth where the orbiting satellite appears stationary to an observer on the Earth's surface.

They have a period of 1 day, orbit around the Earth from west to east and must orbit in the place of the Earth's equator.

Energy of Satellite

Using $\frac{GMm}{r^2} = \frac{mv^2}{r}$, KE of satellite $= \frac{1}{2}mv^2 = \frac{GMm}{2r}$

Given potential energy of satellite is $= -\frac{GMm}{r}$,

total energy E_T of satellite $= -\frac{GMm}{2r}$