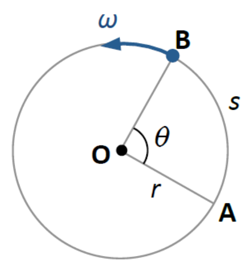
6 Motion in a Circle

Kinematics of Uniform Circular Motion

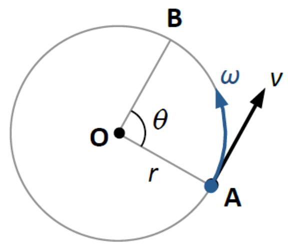


If a particle moves from Point A to B around a fixed point O, the angle ϑ is the **angular displacement**, and is given by $\vartheta = \frac{\text{arc length AB}}{\text{radius OA}}$ which = $\frac{s}{r}$ (dimensionless), or $s = r\vartheta$, where ϑ is measured in radians. **One radian** is thus the angle subtended by an arc length equal to the radius of the arc.

The **angular velocity ω** about O is defined as the rate of change in angular displacement, i.e

$\omega = \frac{d\theta}{dt}$ (rad s^{-1})

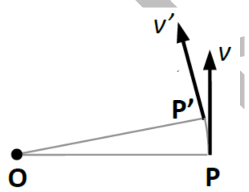
Since 2π = 360° (i.e. one revolution), and the time taken to complete one revolution is known as the period T, $\omega = \frac{2\pi}{T} = 2\pi f$, where $f = \frac{1}{T}$.



To then find the **linear (tangential) velocity *v***, we divide both sides by time and obtain

$v = \frac{s}{t} = \frac{r\theta}{t} = r\omega$ (m s^{-1})

Centripetal Acceleration



A particle moving in a circle with a steady speed v has acceleration as its direction is constantly changing. As the particle moves, its velocity changes from v to v'. The change in velocity Δv is directed towards O for a short time period. Hence, the **centripetal acceleration** (rate of change of velocity) is always <u>directed radially inwards</u>.

Centripetal Acceleration

Centripetal acceleration a_c is given by

$a_c = v\omega = \frac{v^2}{r} = r\omega^2$ (m s^{-2})

Centripetal Force

Since $F = ma$, **centripetal force F_c** is

$F_c = ma_c = mv\omega = m\frac{v^2}{r} = mr\omega^2$ (N)

F_c is directed towards the centre of circular motion, as is centripetal acceleration.

Horizontal Circular Motion

If there is horizontal circular motion, $\Sigma F_x = ma_c$.
If there is no vertical acceleration, $\Sigma F_y = 0$.

Examples:

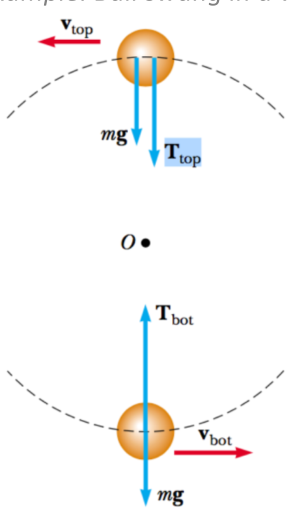
- Unbanked and banked turns

- Pendulum bob

Vertical Circular Motion

Resolve forces acting towards or away from centre of the circle. Resultant provides F_c.
For forces tangential to circle, resultant force provides tangential acceleration.

Example: Ball swung in a vertical circle



$$T_{top} + mg = m\frac{v_{top}^2}{r}$$

$$T_{bot} - mg = m\frac{v_{bot}^2}{r}$$