5 Work, Energy and Power

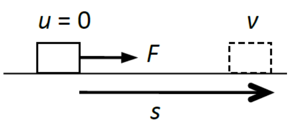
Work

The **work done** (WD) by a force on an object is defined as the product of the force and the object's displacement in the direction of the force, i.e. $W = F \times d \cos\vartheta$ (unit: J) and is a scalar. For a variable force, the WD on an object is the area under the force-displacement graph, i.e. $W = \int F dx$

Potential Energy and Kinetic Energy

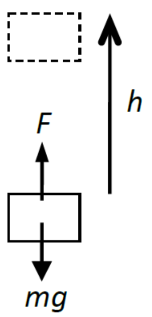
WD on an object leads to a change in energy, thus the different forms of energy can be derived.

Derivation of Kinetic Energy



From $v^2 = u^2 + 2as$, $s = \frac{v^2 - u^2}{2a}$. Using this and $F = ma$, $WD = Fs = ma(\frac{v^2 - u^2}{2a}) = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$, which can be seen as final KE – initial KE. Thus, **kinetic energy** $E_K = \frac{1}{2}mv^2$.

Derivation of Gravitational Potential Energy



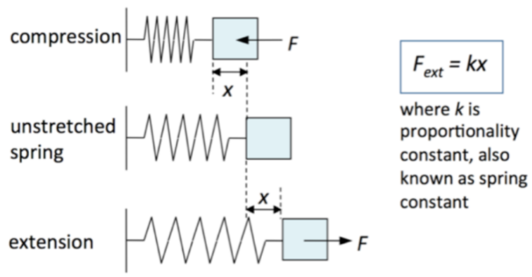
Acceleration of an object raised a distance *h* at constant speed = 0, thus *F* exerted = weight of the object, i.e. $F = mg$. $WD = Fs = mgh$. All WD goes into gain in **gravitational potential energy** E_P, since $\Delta E_K = 0$. Therefore, $E_P = mgh$.

Derivation of Elastic Potential Energy (EPE)

In order to stretch or compress a spring that obeys Hooke's Law, external force must be applied, where $F_{ext} = kx$. WD by external agent = EPE stored = Area under *F*-*x* graph = $\frac{1}{2}kx^2$.

Potential Energy and Kinetic Energy

Derivation of EPE



Generally, any type of potential energy *U* is related to force *F* by $F = -\frac{dU}{dx}$

Energy Conversion and Conservation

Energy (E) is defined as the capacity to do work. Like work, it is a scalar and has the unit Joule J. The **Principle of Energy Conservation** states that the total energy (TE) of a closed system is always constant. Energy can only be converted from one form to another.

By conservation of energy,
$KE_i + PE_i + WD_{ext} = KE_f + PE_f$ of system.
* When accounting for WD by resistive forces, WD has to be negative.
Alternatively, use $\sum$ *energy losses* + WD_{ext} = $\sum$ *energy gain*, where energy loss is calculated by taking $E_{initial} - E_{final}$

Power

Power is the rate of work done and is a scalar, i.e. $P = \frac{d(WD)}{dt}$. Average power $P_{ave} = \frac{Total\ WD}{time\ taken}$. If a constant force *F* acts on an object at velocity *v*, instantaneous power $P = F \times v$.

Due to WD against frictional forces, energy conversion in machines is never 100%. The efficiency of a machine is defined as

$$\text{efficiency} = \frac{\text{useful energy/power output}}{\text{total energy/power input}} \times 100\%$$