12 Superposition

Principle of Superposition

The **principle of superposition** states that when two or more waves are moving through a medium, the resultant displacement at a point is the vector sum of the displacement from the individual waves.

Stationary Waves

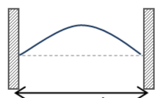
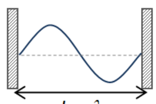
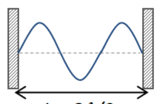
A stationary (or standing) wave is a wave in which the positions of wave elements oscillating with maximum amplitudes (antinodes) and minimum amplitudes (nodes) are fixed with time. It is **formed** as a result of the superposition of two waves of equal frequency and amplitude travelling at the same speed but in opposite directions.

Progressive vs Stationary Waves

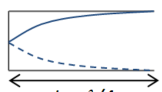
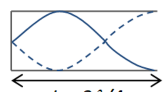
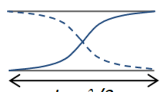
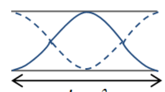
Progressive Wave	**Stationary Wave**
Frequency (same for both)	
All particles in SHM with same frequency	Except nodes, all particles in SHM with frequency = that of component wave
Wavelength (same for both)	
Shortest distance between two in-phase particles	Twice the distance between two adjacent nodes or antinodes
Amplitude (different)	
Same amplitude for all particles	Amplitudes vary from zero for node particles to maximum for antinode particles
Phase relationship (different)	
All particles within 1 wavelength have different phases, given by $\phi = 2\pi\frac{\Delta x}{\lambda}$	Within one loop all particles are in phase. Particles of adjacent loops are π out of phase.
Energy (different)	
Transferred in direction of wave propagation	Energy stored, with antinode particles having most energy.

Progressive vs Stationary Waves

For a string of **fixed length**:

Fundamental (1^{st} Harmonic)	First Overtone (2^{nd} Harmonic)	Second Overtone (3^{rd} Harmonic)
 $L = \lambda/2$	 $L = \lambda$	 $L = 3\lambda/2$
$f_1 = v/(2L)$	$f_2 = v/(L) = 2f_1$	$f_3 = 3v/(2L) = 3f_1$
For n^{th} vibration mode: $L = n(0.5\lambda)$ and $f_n = nf_1$		

For air column of **fixed length**:

Closed pipe	 $L \approx \lambda/4$	 $L \approx 3\lambda/4$
For n^{th} vibration mode: $L = (2n-1)\frac{\lambda}{4}$ & $f_n = (2n-1)f_1$		
Open pipe	 $L \approx \lambda/2$	 $L \approx \lambda$
For n^{th} vibration mode: $L = n\frac{\lambda}{2}$ and $f_n = nf_1$		

Diffraction

Diffraction is only significant and observable when the width of the slit is of the same order of magnitude as the wavelength of the waves, and is defined as the **spreading of waves** when they pass through apertures or around obstacles.

Two-source Interference Patterns

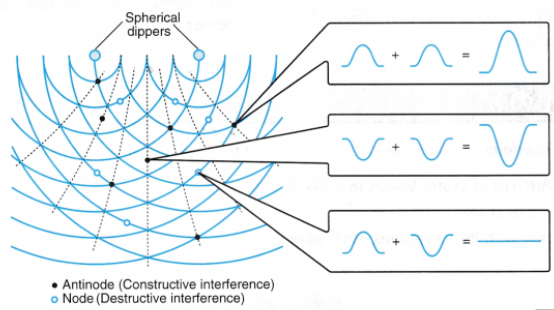
Interference refers to the superposition of two or more waves of the same type passing through the same region of space at a given time. At points where waves arrive in phase/ π out of phase, constructive/destructive interference occurs.

Two-source Interference Patterns

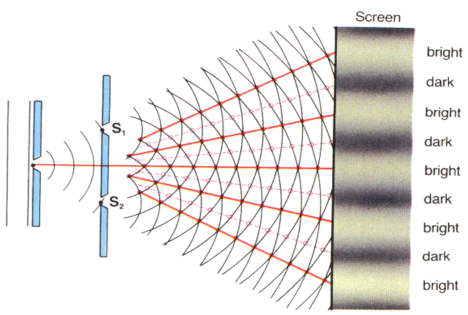
Conditions for observable interference patterns

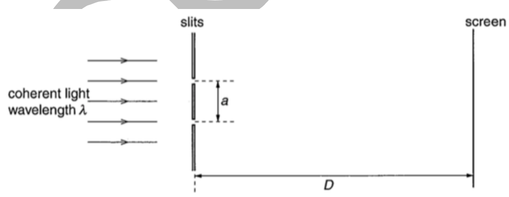
1. Coherent sources are necessary for interference patterns to be observed, i.e. constant phase difference.
2. The two waves must superpose.
3. The two waves must have approximately the same amplitude for sufficient contrast.
4. For transverse waves, they must be either unpolarised or polarised in the same plane.

Water Waves



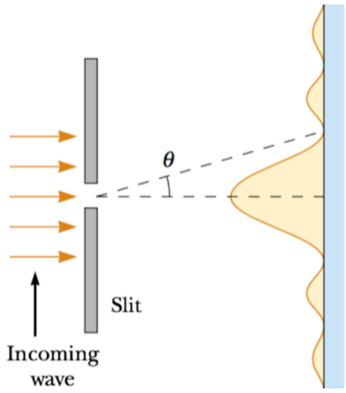
Light Waves (Young's Double Slit)





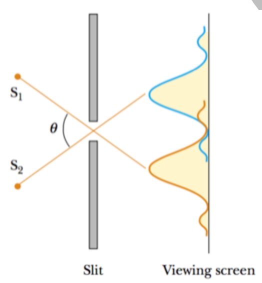
If **a << D**, alternating regions of bright and dark fringes are observed, and fringe separation $x = \frac{\lambda D}{a}$ where a is the slit separation, D is the distance between screen and λ is light wavelength.

Single Slit Diffraction

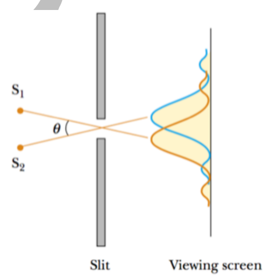


A diffraction pattern is observed as light rays from one portion of the slit interfere with light rays from another portion. For slit width b, the general condition for destructive interference is given by $\sin\theta_{dark} = m\frac{\lambda}{b}$.

Resolution



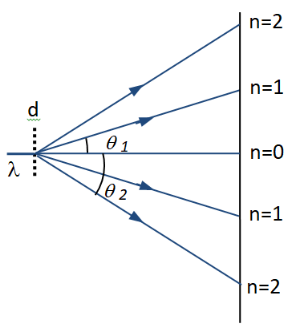
(resolved)



(not resolved)

Rayleigh's criterion sets the limiting condition of resolution, where the central maxima of one object falls on the first minimum of another object. This occurs when the angular separation ϑ subtended by the sources at the slit satisfy $\sin\theta = \frac{\lambda}{b}$. Limiting angle of resolution for a slit of width b is $\vartheta = \frac{\lambda}{b}$.

Diffraction Grating



When a diffraction grating is illuminated normally with a monochromatic light, there will be an interference pattern, where the bright fringes occur for $d\sin\theta = n\lambda$.